

IV Semester B.A./B.Sc. Examination, May/June 2018
(CBCS) (Fresh + Repeaters) (2015 – 16 and Onwards)
(Semester Scheme)
MATHEMATICS (Paper – IV)

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all Parts.**PART – A**1. Answer **any five** questions.**(5×2=10)**

- a) Define a normal subgroup.
- b) If $f : G \rightarrow G'$ is a homomorphism then prove that $f(e) = e'$, where e and e' are the identity elements of G and G' respectively.
- c) Expand $f(x) = x$ in half range cosine series over the interval $(0, \pi)$.
- d) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at $(1, 1)$.
- e) If $L[f(t)] = F(s)$, then show that $L[e^{at} f(t)] = F(s - a)$.
- f) Find $L[t \sin t]$.
- g) Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.
- h) Prove that 'x' is a part of the complementary function of

$$x^2 \frac{d^2y}{dx^2} - 2x(x+1) \frac{dy}{dx} + 2(x+1)y = x^3.$$

PART – BAnswer **one full** question.**(1×15=15)**

2. a) Prove that a subgroup H of a group G is normal subgroup of G iff $gHg^{-1} = H, \forall g \in G$.
- b) Define centre of a group and prove that the centre of a group G is a normal subgroup of G .
- c) If $f : G \rightarrow G'$ be a homomorphism from the group G into G' with Kernel K , then prove that f is one-one iff $K = \{e\}$, where 'e' is the identity element in G .

OR



3. a) Prove that the intersection of two normal subgroups of a group is a normal subgroup.
- b) If G is a group and H is a subgroup of index 2 in G , then show that H is a normal subgroup of G .
- c) State and prove Fundamental theorem of Homomorphism.

PART - C

(2x15=30)

Answer two full questions.

4. a) Find the Fourier expansion of $f(x) = x - x^2$ in $(-1, 1)$.
- b) Obtain half range sine series of $f(x) = \sin x$, $0 < x < \pi$.
- c) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ by Taylor series upto 3rd degree terms.

OR

5. a) Find the Fourier series of $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ \pi - x, & \pi \leq x \leq 2\pi \end{cases}$.
- b) Find the extreme values of the function $f(x) = x^3y^2(1 - x - y)$.
- c) Show that minimum value of $x^2 + y^2 + z^2$ subjected to the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ is 27.

6. a) Find $L[e^t \sin^2 t]$ and $L[\sinh^2 at]$.

b) Find $L[f(t)]$ if $f(t) = \begin{cases} 2t, & 0 \leq t \leq 5 \\ 1, & t > 5 \end{cases}$.

c) Using Convolution theorem find $L^{-1} \left[\frac{1}{(s+2)(s+4)} \right]$.

OR

7. a) Find :

i) $L[t^3 e^{-3t}]$.

ii) $L[e^{-t}(2 \cos 5t - 3 \sin 5t)]$.

b) Find $L^{-1} \left[\frac{s^2}{(s-1)(s^2+1)} \right]$.

c) Find $L[t^2 u(t-3)]$ using convolution property.

PART - D

(1×15=15)

Answer **one full** question.

8. a) Solve : $(D^2 - 5D + 6)y = e^{4x} + \sin 2x$.

b) Solve : $4x^2 y'' + 4xy' - y = 4x^2$.

c) Solve : $xy'' - (1+x)y' + y = 0$, given that $(x+1)$ is a part of complementary function.

OR

9. a) Solve : $\frac{d^2y}{dx^2} + y = e^{-x} + 5x^2 e^x$.

b) Solve : $\frac{dx}{dt} = 3x - y$; $\frac{dy}{dt} = x + y$.

c) Solve : $y'' + y = \tan x$ by the method of variation of parameters.